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The equivalence theorem for gauge boson scattering in a five-dimensional Standard Model

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Abstract

We present an equivalence theorem for the longitudinal components of the gauge bosons in a compactified five-dimensional extension of the Standard Model, whose spontaneous symmetry breaking is driven either by one Higgs in the bulk or by one on a brane or by both together. We also show some implications for the unitarity bounds on Higgs masses.

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1. Introduction

There has been recently a growing interest in theories with large and extra large dimensions motivated by the multidimensional unification of gravitational, strong and electroweak interactions through string theory. Special attention has been devoted to the brane picture where ordinary matter lives in four dimensions while gravity propagates in the bulk. Specific models relating the solution to the hierarchy problem to the existence of a large volume for the n extra dimensions [1] or to an exponential warp factor in a five-dimensional (5D) non-factorizable metric [2] have been suggested; as in all Kaluza–Klein (KK) theories the compactification process produce a tower of graviton and scalar excitations, whose phenomenology has been studied in [3].

In addition there are realizations where also the gauge interactions feel some extra dimensions, parallel to the brane: supersymmetric 5D Standard Model (SM) extensions have been proposed, where the supersymmetry breaking scale is related to the compactification scale which, therefore, turns out to be in the TeV range [4]. Many formal and phenomenological aspects of these models have been investigated; in particular these models contain KK towers of excitations of the W , Z gauge bosons, of the photon and possibly of the Higgs. Lower bounds from the electroweak precision data on the compactification scale of these models, when fermions are localized on the brane or in different points of the bulk, are in the range of 2–5 TeV [5,6]. These bounds become much weaker, 300–400 GeV, when all particles live in the bulk [7].

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Recently the unitarity of 5D Yang–Mills theories has been considered [10], proving a theorem similar to the standard equivalence theorem (ET) [11,12] that relates at high energies the longitudinal components of gauge bosons to their associated Goldstone bosons (GB). In the unbroken extra-dimensional Yang–Mills case, what has been shown is the equivalence of longitudinal KK gauge bosons $V_{(n)}^\mu$ and their corresponding $V_{(n)}^5$ components of the 5D gauge fields.

The aim of this Letter is to show this equivalence in the case of spontaneously broken 5D extensions of the SM. The main subtlety in the proof is that, the usual SM would-be GB can mix with KK states. In Section 2, using the formalism of the non-supersymmetric 5D SM and its compactification to four dimensions, we identify those GB and $V_{(n)}^5$ KK combinations $\widehat{G}_{(n)}^V$ that couple to the gauge field derivatives in the gauge fixing term. From here proceeds the standard proof of the ET in the R_ξ gauge that relates the scattering amplitudes between longitudinal KK gauge bosons and those of their corresponding $\widehat{G}_{(n)}^V: T(\widehat{V}_{L(m)}^\mu, \widehat{V}_{L(n)}^\mu, \dots) \simeq T(\widehat{G}_{(m)}^V, \widehat{G}_{(n)}^V, \dots) + O(M_k/E)$, M_k being the biggest one of the masses of the KK gauge bosons. Finally, we illustrate the use of the ET to calculate the $\widehat{W}_{L(m)}^+ \widehat{W}_{L(n)}^- \rightarrow \widehat{W}_{L(p)}^+ \widehat{W}_{L(q)}^-$ scattering amplitudes for the channel. These amplitudes are relevant to investigate how the partial wave unitarity limit on the mass of the Higgs is modified by extra dimensions. Some aspects of this problem have been addressed in particular by studying the effect of the radion in the HH [8] or in the $W_L^+ W_L^-$ amplitudes [9]. In Section 3 we calculate the amplitudes in the case with just one Higgs in the bulk, in Section 4, with just one Higgs on the brane and finally, in Section 5 we study the general case with one Higgs on the brane and another one in the bulk.

2. The equivalence theorem for 5D fields

We consider a minimal 5D extension of the SM with two scalar fields, compactified on the segment S^1/Z_2 , of length πR , in which the $SU(2)_L$ and $U(1)_Y$ gauge fields and the Higgs field Φ_1 propagate in the bulk while the Higgs field Φ_2 lives on the brane at $y = 0$. The Lagrangian of the gauge Higgs sector is given by (see [6] for a review)

$$\begin{aligned} & \int_0^{2\pi R} dy \int dx \mathcal{L}(x, y) \\ &= \int_0^{2\pi R} dy \int dx \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} F_{MN}^a F^{aMN} + \mathcal{L}_{\text{GF}}(x, y) \right. \\ & \quad \left. + (D_M \Phi_1)^\dagger (D^M \Phi_1) + \delta(y) (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V(\Phi_1, \Phi_2) \right\}, \end{aligned} \quad (1)$$

where $M = \mu, 5$, B_{MN} , F_{MN}^a are the $U(1)_Y$ and $SU(2)_L$ field strengths and a is the $SU(2)$ index. Note that Φ_1 has energy dimension $3/2$, whereas Φ_2 has dimension 1. The covariant derivative is defined as $D_M = \partial_M - ig_5 A_M^a \tau^a/2 - ig_5' B_M/2$. For simplicity we will consider a Higgs potential symmetric under the discrete symmetry $\Phi_2 \rightarrow -\Phi_2$, which is given by

$$\begin{aligned} V(\Phi_1, \Phi_2) &= \mu_1^2 (\Phi_1^\dagger \Phi_1) + \lambda_1^{(5)} (\Phi_1^\dagger \Phi_1)^2 \\ &+ \delta(y) \left[\frac{1}{2} \mu_2^2 (\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \frac{1}{2} \lambda_3^{(5)} (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \right. \\ & \quad \left. + \frac{1}{2} \lambda_4^{(5)} (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \lambda_5^{(5)} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right], \end{aligned} \quad (2)$$

where the dimensionalities of these couplings are: 1 for μ_1 and μ_2 , -1 for $\lambda_1^{(5)}, \lambda_3^{(5)}, \lambda_4^{(5)}$ and $\lambda_5^{(5)}$, whereas λ_2 is dimensionless. Also for simplicity, we will require $\lambda_3^{(5)} + \lambda_4^{(5)} + 2\lambda_5^{(5)} = 0$, which ensures that the minimum of the potential corresponds to the constant configuration $\Phi_1 = (0, v_1/\sqrt{4\pi R})$, $\Phi_2 = (0, v_2/\sqrt{2})$, where $v_1^2 \equiv -2\pi R\mu_1^2/\lambda_1^{(5)}$ and $v_2^2 \equiv -\mu_2^2/\lambda_2$. In this way, the Higgs fields are expanded in the standard form

$$\Phi_1(x, y) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\omega^1 - i\omega^2) \\ \frac{1}{\sqrt{2}}\left(\frac{v_1}{\sqrt{2\pi R}} + h_1 - i\omega^3\right) \end{pmatrix}, \quad \Phi_2(x) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\pi^1 - i\pi^2) \\ \frac{1}{\sqrt{2}}(v_2 + h_2 - i\pi^3) \end{pmatrix}, \quad (3)$$

where following the standard two Higgs notations $v_1 = v \cos \beta$, $v_2 = v \sin \beta$ are the vacuum expectation values of the scalar fields and $v^2 = (\sqrt{2} G_F)^{-1}$. For brevity we will use the notation $c_\beta \equiv \cos \beta$, $s_\beta \equiv \sin \beta$.

The gauge fixing Lagrangian $\mathcal{L}_{\text{GF}}(x, y)$ is

$$\begin{aligned} \mathcal{L}_{\text{GF}}(x, y) &= -\frac{1}{2\xi}(F^a(A^a))^2 - \frac{1}{2\xi}(F(B))^2, \\ F^a(A^a) &= \partial_\mu A^{a\mu} - \xi \left[\partial_5 A_5^a - \frac{g_5 v c_\beta}{2\sqrt{2\pi R}} \omega^a - \frac{g_5 v s_\beta}{2} \pi^a \delta(y) \right], \\ F(B) &= \partial_\mu B^\mu - \xi \left[\partial_5 B_5 + \frac{g'_5 v c_\beta}{2\sqrt{2\pi R}} \omega^3 + \frac{g'_5 v s_\beta}{2} \pi^3 \delta(y) \right], \end{aligned} \quad (4)$$

where, in order to avoid a gauge dependent mixing angle between the physical Z and the photon, we have chosen the same ξ parameter for the $A^{a\mu}$ and B^μ fields. Let us now recall that the fields living in the bulk have a Fourier expansion, which is:

$$X(x, y) = \frac{1}{\sqrt{2\pi R}} X_{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \cos\left(\frac{ny}{R}\right) X_{(n)}(x), \quad (5)$$

for $X = A_\mu^a, B_\mu, \omega^a, h_1$, whereas for $Y = A_5^a, B_5$ it is

$$Y(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \sin\left(\frac{ny}{R}\right) Y_{(n)}(x). \quad (6)$$

Note that the condition $\lambda_3^{(5)} + \lambda_4^{(5)} + 2\lambda_5^{(5)} = 0$ yields a diagonal Higgs mass matrix: $m_{h_1(0)}^2 = 2v_1^2\lambda_1$, $m_{h_2}^2 = 2v_2^2\lambda_2$, $m_{h_1(n)}^2 = 2v_1^2\lambda_1 + n^2/R^2$, where $\lambda_1 = \lambda_1^{(5)}/(2\pi R)$.

Similarly to the SM case in four dimensions, we define the following charged and neutral field combinations $W_M^\pm = (A_M^1 \mp iA_M^2)/\sqrt{2}$, $Z_M = (g_5 A_M^3 - g'_5 B_M)/\sqrt{g_5^2 + g'^2}$, $A_M = (g'_5 A_M^3 + g_5 B_M)/\sqrt{g_5^2 + g'^2}$. After integrating out the compactified fifth dimension y , the mass matrix \mathcal{M}_V^2 of the gauge bosons and their KK excitations has the following $(N+1) \times (N+1)$ generic form (with $N \rightarrow \infty$):

$$(V_{(0)}, V_{(1)}, V_{(2)}, \dots) \begin{pmatrix} m^2 + d_0^2 & \sqrt{2}m^2 & \sqrt{2}m^2 & \dots \\ \sqrt{2}m^2 & 2m^2 + d_1^2 & \sqrt{2}m^2 & \dots \\ \sqrt{2}m^2 & \sqrt{2}m^2 & 2m^2 + d_2^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} V_{(0)} \\ V_{(1)} \\ V_{(2)} \\ \vdots \end{pmatrix}, \quad (7)$$

where $m^2 = m_V^2 s_\beta^2$ and $d_0 = m_V c_\beta$, $d_n = \sqrt{(n/R)^2 + d_0^2}$. In particular, for $V = W_\mu^\pm$, $m_W = gv/2$, whereas for $V = Z_\mu$, $m_Z = \sqrt{g^2 + g'^2} v/2$, with $g = g_5/\sqrt{2\pi R}$ and $g' = g'_5/\sqrt{2\pi R}$. Note that for the photon $m = m_A = 0$,

the mass matrix is already diagonal, the photon has zero mass and for its associated KK states the masses are given by $m_{A(n)} = n/R$.

For the $V = W_\mu^\pm, Z_\mu$ case, \mathcal{M}_V^2 is diagonal when $s_\beta = 0$ and when $s_\beta \neq 0$ it has the following eigenvalue equation

$$\sqrt{m_{V(n)}^2 - d_0^2} = \frac{m^2}{\sqrt{m_{V(n)}^2 - d_0^2}} \left(1 + 2 \sum_{i=1}^N \frac{m_{V(n)}^2 - d_0^2}{m_{V(n)}^2 - d_i^2} \right) \xrightarrow{N \rightarrow \infty} \pi m^2 R \cot(\pi R \sqrt{m_{V(n)}^2 - d_0^2}), \quad (8)$$

so that it can be diagonalized with $P_V^t \mathcal{M}_V^2 P_V = \text{diag}\{m_{V(0)}^2, m_{V(1)}^2, \dots\}$, where

$$P_V = \begin{pmatrix} \frac{u_0}{\sqrt{2}} & \cdots & \frac{u_n}{\sqrt{2}} & \cdots \\ \frac{u_0(d_0^2 - m_{V(0)}^2)}{d_1^2 - m_{V(0)}^2} & \cdots & \frac{u_n(d_0^2 - m_{V(n)}^2)}{d_1^2 - m_{V(n)}^2} & \cdots \\ \vdots & \ddots & \vdots & \vdots \\ \frac{u_0(d_0^2 - m_{V(0)}^2)}{d_n^2 - m_{V(0)}^2} & \cdots & \frac{u_n(d_0^2 - m_{V(n)}^2)}{d_n^2 - m_{V(n)}^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (9)$$

is a $(N+1) \times (N+1)$ matrix and

$$u_j = \sqrt{\frac{1}{2} + \sum_{i=1}^N \left[\frac{d_0^2 - m_{V(j)}^2}{d_i^2 - m_{V(j)}^2} \right]^2} = \frac{2m^2}{\sqrt{m_{V(j)}^2 - d_0^2} \sqrt{m^2(1 + \pi^2 R^2 m^2) + m_{V(j)}^2 - d_0^2}}, \quad (10)$$

where we have used, in the $N \rightarrow \infty$ limit, the following series

$$\sum_{i=1}^{\infty} \frac{(d_0^2 - m_{V(j)}^2)^2}{(d_i^2 - m_{V(j)}^2)^2} = \frac{1}{4} \left[-2 + \frac{m_{V(j)}^2 - d_0^2}{m^2} + \pi^2 R^2 (m_{V(j)}^2 - d_0^2) + \frac{(m_{V(j)}^2 - d_0^2)^2}{m^4} \right]. \quad (11)$$

Thus, the gauge boson mass eigenstates are $\hat{V}_{(n)} = (P_V)_{nm} V_{(m)}$.

After integrating out the fifth dimension, and by separating the charged and neutral field combinations, the gauge fixing conditions in Eq. (4) become

$$\begin{aligned} \mathcal{L}_{\text{GF}}(x) = & -\frac{1}{2\xi} \sum_{n=0}^{\infty} \left\{ 2 \left| \partial_\mu W_{(n)}^{+\mu} - \xi \left(\sqrt{\frac{n^2}{R^2} + m_W^2 c_\beta^2} G_{(n)}^+ - \sqrt{2}^{1-\delta_{n,0}} m_W s_\beta \pi^+ \right) \right|^2 \right. \\ & + \left[\partial_\mu Z_{(n)}^\mu - \xi \left(\sqrt{\frac{n^2}{R^2} + m_Z^2 c_\beta^2} G_{(n)}^Z - \sqrt{2}^{1-\delta_{n,0}} m_Z s_\beta \pi^3 \right) \right]^2 \\ & \left. + \left[\partial_\mu A_{(n)}^\mu - \xi \frac{n}{R} A_{(n)}^5 \right]^2 \right\}, \end{aligned}$$

where $\pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \mp i\pi^2)$, $\omega^\pm = \frac{1}{\sqrt{2}}(\omega^1 \mp i\omega^2)$. We have also defined

$$\begin{aligned} G_{(0)}^\pm &= -\omega_{(0)}^\pm, & G_{(n)}^\pm &= c_n^W W_{5(n)}^\pm + s_n^W \omega_{(n)}^\pm, & n \geq 1, \\ G_{(0)}^Z &= -\omega_{(0)}^3, & G_{(n)}^Z &= c_n^Z Z_{5(n)} + s_n^Z \omega_{(n)}^3, & n \geq 1, \end{aligned} \quad (12)$$

where $s_n^V = -m_V c_\beta / \sqrt{n^2/R^2 + m_V^2 c_\beta^2}$ and $c_n^V = (n/R) / \sqrt{n^2/R^2 + m_V^2 c_\beta^2}$. In general, for the calculations of amplitudes we would also need the orthogonal combinations

$$a_{(n)}^\pm = -s_n^W W_{5(n)}^\pm + c_n^W \omega_{(n)}^\pm, \quad a_{(n)}^Z = -s_n^Z Z_{5(n)} + c_n^Z \omega_{(n)}^3, \quad n \geq 1. \quad (13)$$

Note that, as commented in the introduction, the usual GB and their KK excitations are mixed with the KK states of W_5^\pm and Z_5 , in the gauge fixing term.

Once we have written the gauge fixing fields in the charged-neutral basis, in order to find the GB mass eigenstates it is very convenient to rewrite the gauge fixing in a more compact matrix form including all the KK excitations. For the sake of brevity, we gather the gauge bosons in an $(N+1)$ -dimensional vector $\mathbf{V}^\mu = (V_{(0)}^\mu, V_{(1)}^\mu, \dots)^t$, where now $V = W^\pm, Z, A$, whereas the GB and the pseudoscalars are gathered in the $(N+2)$ -dimensional vectors $\mathbf{G}^\pm = (\pi^\pm, G_{(0)}^\pm, G_{(1)}^\pm, \dots)^t$, $\mathbf{G}^Z = (\pi^3, G_{(0)}^Z, G_{(1)}^Z, \dots)^t$, $\mathbf{G}^A = (0, 0, A_{5(1)}, A_{5(2)}, \dots)^t$. With these definitions

$$\mathcal{L}_{\text{GF}}(x) = -\frac{1}{2\xi} \{ 2|\partial_\mu \mathbf{W}^{+\mu} - \xi M_\xi^W \mathbf{G}^+|^2 - |\partial_\mu \mathbf{Z}^\mu - \xi M_\xi^Z \mathbf{G}^Z|^2 - |\partial_\mu \mathbf{A}^\mu - \xi M_\xi^A \mathbf{G}^A|^2 \},$$

the $((N+1) \times (N+2))$ -dimensional M_ξ^V matrix being generically of the form

$$M_{V\xi} = \begin{pmatrix} -m & d_0 & 0 & 0 & \dots \\ -\sqrt{2}m & 0 & d_1 & 0 & \dots \\ -\sqrt{2}m & 0 & 0 & d_2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (14)$$

The gauge-fixing term provides a gauge-dependent mass term for the would-be GB $\mathcal{L}_{\xi \text{mass}} = -\xi \mathbf{G}^+ \mathcal{M}_{W\xi}^2 \mathbf{G}^- - \xi \mathbf{G}^{Zt} \mathcal{M}_{Z\xi}^2 \mathbf{G}^Z / 2 - \xi \mathbf{G}^{At} \mathcal{M}_{A\xi}^2 \mathbf{G}^A / 2$, with

$$\mathcal{M}_{V\xi}^2 \equiv M_{V\xi}^t M_{V\xi} = \begin{pmatrix} m^2(1 + \sum_{i=1}^N 2) & -md_0 & -\sqrt{2}md_1 & \dots \\ -md_0 & d_0^2 & 0 & \dots \\ -\sqrt{2}md_1 & 0 & d_1^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (15)$$

being a $(N+2) \times (N+2)$ matrix, whose eigenvalues are the same as those of \mathcal{M}_V^2 , plus a zero. For the photon that matrix is already diagonal, whereas for the $V = W_\mu^\pm, Z_\mu$ cases, it can be diagonalized to $Q_V^t \mathcal{M}_{V\xi}^2 Q_V = \text{diag}\{0, m_{V(0)}^2, m_{V(1)}^2, \dots\}$ using the $(N+2) \times (N+2)$ orthogonal matrix:

$$Q_V = \begin{pmatrix} q_{-1}/m & q_0/m & q_1/m & \dots \\ \frac{q_{-1}}{d_0} & \frac{d_0 q_0}{d_0^2 - m_{V(0)}^2} & \frac{d_0 q_1}{d_0^2 - m_{V(1)}^2} & \dots \\ \frac{\sqrt{2}q_{-1}}{d_1} & \frac{\sqrt{2}d_1 q_0}{d_1^2 - m_{V(0)}^2} & \frac{\sqrt{2}d_1 q_1}{d_1^2 - m_{V(1)}^2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (16)$$

where

$$q_{-1} = -\left(\frac{1}{m^2} + \sum_{i=0}^N \frac{2^{1-\delta_{i,0}}}{d_i^2}\right)^{-1/2}, \quad q_j = -\left(\frac{1}{m^2} + \sum_{i=0}^N \frac{2^{1-\delta_{i,0}} d_i^2}{(d_i^2 - m_{V(j)}^2)^2}\right)^{-1/2}. \quad (17)$$

Let us remark that the rotations to obtain the gauge field mass eigenstates, P_V , are different from those of the would-be GB, Q_V . Consequently there could be a modification to the ET that relates amplitudes of longitudinal mass eigenstate gauge bosons, $\widehat{V}^\mu = P_V^t \mathbf{V}^\mu$, with those containing would be GB, $\widehat{G}^V = Q_V^t \mathbf{G}^V$ in the R_ξ gauges. As it is well known, the ET follows from the gauge-fixing Lagrangian [11,12], which, in terms of mass eigenstates, is now written as

$$\mathcal{L}_{\text{GF}}(x) = -\frac{1}{2\xi} \{2|\partial_\mu \widehat{W}^{+\mu} - \xi P_W^t M_\xi^W Q_W \widehat{G}^+|^2 + |\partial_\mu \widehat{Z}^\mu - \xi P_Z^t M_\xi^Z Q_Z \widehat{G}^Z|^2 + |\partial_\mu \mathbf{A}^\mu - \xi M_\xi^A \mathbf{G}^A|^2\}. \quad (18)$$

In this way it may seem that the n mode of the gauge field eigenstates $\widehat{V}_{(n)}^\mu$ could mix with all the $\widehat{G}_{(m)}^V$. Amazingly, the q_i are related to the u_i (also for finite N):

$$q_i = \frac{d_0^2 - m_{V(i)}^2}{\sqrt{2} m_{V(i)}} u_i. \quad (19)$$

Which allows us to write:

$$P_V^t M_{V\xi} Q_V = \begin{pmatrix} 0 & m_{V(0)} & 0 & \dots \\ 0 & 0 & m_{V(1)} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (20)$$

and, therefore, there is no mixing between the $N+1$ gauge bosons $\widehat{V}_{(n)}^\mu$ and the $N+2$ GB $\widehat{G}_{(m)}^V$ unless $n=m$. In other words, the longitudinal components of the $\widehat{V}_{(n)}^\mu$ will “eat” only the corresponding $\widehat{G}_{(n)}^V$, which is an eigenstate of the gauge-dependent GB mass matrix. In particular, the $\widehat{G}_{(-1)}^V$ are not GB combinations “eaten” by the longitudinal gauge bosons, but remain in the physical spectrum as the physical charged and neutral pseudoscalars. We can thus write simply:

$$\mathcal{L}_{\text{GF}}(x) = -\frac{1}{\xi} \sum_{n=0}^{\infty} \left\{ \frac{1}{2} \left[\partial_\mu A_{(n)}^\mu - \xi \frac{n}{R} A_{(n)}^5 \right]^2 + |\partial_\mu \widehat{W}_{(n)}^{+\mu} - \xi m_{W(n)} \widehat{G}_{(n)}^+|^2 + \frac{1}{2} [\partial_\mu \widehat{Z}_{(n)}^\mu - \xi m_{Z(n)} \widehat{G}_{(n)}^Z]^2 \right\}. \quad (21)$$

Once identified the $\widehat{G}_{(n)}^V$ fields that couple diagonally with the derivatives of the gauge boson mass eigenstates, the ET proof proceeds as usual [11,12], simply by substituting $V_L \rightarrow \widehat{V}_L$ and the would-be GB by \widehat{G}^V . Therefore, we arrive at

$$T(\widehat{V}_{L(m)}^\mu, \widehat{V}_{L(n)}^\mu, \dots) \simeq C^{(m)} C^{(n)} \dots T(\widehat{G}_{(m)}^V, \widehat{G}_{(n)}^V, \dots) + O(M_k/E) \quad (22)$$

M_k being the biggest one of the $m_{V(m)}, m_{V(n)}, \dots$ masses, and the $C^{(i)} = 1 + O(g)$ account for renormalization corrections (see the last three references in [11]).

3. The 5D SM with one Higgs on the brane

Let us study first the simple case of a single Higgs on the brane, which is obtained from the general case by taking the $c_\beta, \lambda_i^{(5)} \rightarrow 0, i=1, 3, 4, 5, \mu_1 \rightarrow \infty$ limit. As an application of the ET we will illustrate how to calculate the $\widehat{W}_{L(m)}^+ \widehat{W}_{L(n)}^- \rightarrow \widehat{W}_{L(p)}^+ \widehat{W}_{L(q)}^-$ amplitudes, with $m, n, p, q \geq 0$, which are thus related to $T_{mnpq} = \widehat{G}_{(m)}^+ \widehat{G}_{(n)}^- \rightarrow \widehat{G}_{(p)}^+ \widehat{G}_{(q)}^-$. Among other things, these amplitudes are interesting to obtain bounds on the Higgs masses from tree level unitarity. Similarly to what it is done to obtain the unitarity limits in the SM, we are only interested in the lowest order calculation in the gauge couplings g and g' .

We have decoupled the ω fields, so that $G_{(0)}^\pm \rightarrow 0$ and $\mathbf{G}^\pm = (\pi^\pm, 0, W_{5(1)}^\pm, W_{5(2)}^\pm, \dots)^t$. Moreover, since $\widehat{\mathbf{G}}^\pm = Q_W^t \mathbf{G}^\pm$, and in this case $d_0 \rightarrow 0$, $q_{(-1)}/d_0 \rightarrow -1$ so that

$$\widehat{G}_{(-1)}^\pm \rightarrow 0, \quad \widehat{G}_{(i)}^\pm = q_i \left(\frac{1}{m_W} \pi^\pm + \sqrt{2} \sum_{n=1}^N \frac{n/R}{(n/R)^2 - m_{W(i)}^2} W_{5(n)}^\pm \right), \quad i \geq 0. \quad (23)$$

Furthermore the scalar potential now only depends on Φ_2 . After integration on the 5th dimension, the relevant coupling terms for the amplitude above are the usual $\lambda_2(\pi^+\pi^-)^2 + 2\lambda_2 v_2 \pi^+\pi^- h_2$. Note that in this case it is enough to look for couplings of the π fields since there is no coupling of $W_{5(n)}^\pm$ gauge field components to the Φ_2 scalar sector in $(D_\mu \Phi_2)^\dagger D^\mu \Phi_2$, but only a mixing from the gauge fixing. Thus by substituting $\pi^\pm = \sum_{i=0}^N (q_i/m_W) \widehat{G}_{(i)}^\pm$ in the coupling terms, we find, for $\sqrt{s} \gg m_{W(m)}, m_{W(n)}, m_{W(p)}, m_{W(q)}$

$$T_{mnpq} = -i\sqrt{2} G_F m_{h_2}^2 \frac{q_m}{m_W} \frac{q_n}{m_W} \frac{q_p}{m_W} \frac{q_q}{m_W} \left(\frac{s}{s - m_{h_2}^2} + \frac{t}{t - m_{h_2}^2} \right). \quad (24)$$

In particular, for the scattering of longitudinal zero modes, we find the very same SM amplitude [12], but corrected by a factor

$$\left(\frac{q_0}{m_W} \right)^4 = \left(\frac{2}{(1 + \pi^2 R^2 m_W^2) + m_{W(0)}^2/m_W^2} \right)^2 \simeq 1 - \frac{2}{3} m_W^2 \pi^2 R^2 + O(m_W^4 R^4), \quad (25)$$

where we have used Eqs. (19) and (10). In the last step we have also used the small R approximation $m_{W(0)}^2 \simeq m_W^2 (1 - \pi^2 R^2 m_W^2/3)$ obtained by expanding the $c_\beta \rightarrow 0$ limit of the eigenvalue equation in Eq. (8). As long as $R > (3 \text{ TeV})^{-1}$, the corrections are rather small: $O(m_W^2 R^2) \simeq 10^{-3}$.

Nevertheless, we next show that the modification from the four gauge boson amplitudes can be even smaller. As a matter of fact, the complete study of the unitarity bounds involves amplitudes also with the Higgs or the W^\pm and Z gauge bosons. In particular, one is interested in the largest eigenvalue of the matrix made of all these amplitudes. The complete analysis of the unitarity bounds lies, therefore, beyond our applications of the ET. However, the ET will allow us to calculate the block of T_{mnpq} amplitudes in the $s, t \rightarrow \infty$ limit

$$\begin{pmatrix} T_{0000} & T_{0010} & T_{0001} & T_{0011} & \cdots \\ T_{1000} & T_{1010} & T_{1001} & T_{1011} & \cdots \\ T_{0100} & T_{0110} & T_{0101} & T_{0111} & \cdots \\ T_{1100} & T_{1110} & T_{1101} & T_{1111} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \xrightarrow{s \rightarrow \infty} -i \frac{4m_{h_2}^2 G_F}{\sqrt{2} m_W^4} \begin{pmatrix} q_0^4 & q_0^3 q_1 & q_0^2 q_1^2 & q_0^2 q_1^2 & \cdots \\ q_0^3 q_1 & q_0^2 q_1^2 & q_0^2 q_1^2 & q_0 q_1^3 & \cdots \\ q_0^3 q_1 & q_0^2 q_1^2 & q_0^2 q_1^2 & q_0 q_1^3 & \cdots \\ q_0^2 q_1^2 & q_0 q_1^3 & q_0 q_1^3 & q_1^4 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (26)$$

It can be shown that the largest eigenvalue is $-i4m_{h_2}^2 G_F (q_0^2 + q_1^2 + q_2^2 + \dots)/(\sqrt{2} m_W^4)$ and, therefore, the SM value is modified by a factor

$$\begin{aligned} \left(\sum_{k=0}^{\infty} \frac{q_k^2}{m_W^2} \right)^2 &= \left(\frac{q_0}{m_W} \right)^4 + 2 \left(\frac{q_0}{m_W} \right)^2 \frac{1}{m_W^2} \sum_{k=1}^{\infty} q_k^2 + \cdots \\ &\simeq 1 - \frac{2}{3} m_W^2 \pi^2 R^2 + 2 \sum_{k=1}^{\infty} \frac{2}{1 + \pi^2 R^2 m_W^2 + k^2/(m_W^2 R^2)} + O(m_W^4 R^4) \\ &\simeq 1 - \frac{2}{3} m_W^2 \pi^2 R^2 + 4m_W^2 R^2 \sum_{k=1}^{\infty} \frac{1}{k^2} + O(m_W^4 R^4) \simeq 1 + O(m_W^4 R^4), \end{aligned} \quad (27)$$

where we have approximated $m_{W(k)} \simeq k/R$ for $k \geq 1$ and small R .

We stress that, as long as $s \gg m_{W(n)}^2, m_{Z(n)}^2$, the same matrix pattern of Eq. (26) occurs for any other four-gauge boson amplitude matrices, and therefore, at least in the gauge sector, the same strong cancellations up to $O(m_W^4 R^4)$ will occur.

4. The 5D SM with one Higgs in the bulk

Let us then study the other limiting case when there is only one Higgs in the bulk, which is obtained from the general case by taking the $s_\beta, \lambda_2, \lambda_i^{(5)} \rightarrow 0, i = 3, 4, 5, \mu_2 \rightarrow \infty$ limit. Note that $m = 0$ and, therefore, all the \mathcal{M}_V^2 and $\mathcal{M}_{V\xi}^2$ matrices are already diagonal. Physically, this means that there is no mixing between any KK mode of different KK level. Thus, everything is simpler since $\widehat{V} = V$ and $\widehat{G}^V = G^V$, with $c_\beta = 1$ in Eq. (12).

As before, we will calculate the $\widehat{W}_{L(m)}^+ \widehat{W}_{L(n)}^- \rightarrow \widehat{W}_{L(p)}^+ \widehat{W}_{L(q)}^-$ amplitudes, with $m, n, p, q \geq 0$, which, at high energies, are related through the ET with $T_{mnpq} = G_{(m)}^+ G_{(n)}^- \rightarrow G_{(p)}^+ G_{(q)}^-$.

Once more we are only interested in the lowest order calculation in g and g' and thus we do not need the $\omega\omega V$ couplings. Therefore, the only relevant interactions come from the scalar potential and are given by

$$\begin{aligned} \lambda_1 \Big\{ & (\omega_{(0)}^- \omega_{(0)}^+)^2 + \omega_{(0)}^- \omega_{(0)}^- \omega_{(n)}^+ \omega_{(n)}^+ + \omega_{(0)}^+ \omega_{(0)}^+ \omega_{(n)}^- \omega_{(n)}^- + 4\omega_{(0)}^- \omega_{(0)}^+ \omega_{(n)}^- \omega_{(n)}^+ \\ & + \sqrt{2}(\omega_{(0)}^- \omega_{(n)}^+ \omega_{(m)}^- \omega_{(p)}^+ + \text{h.c.}) \Delta_3(m, n, p) + \frac{1}{2} \omega_{(n)}^- \omega_{(m)}^+ \omega_{(p)}^- \omega_{(q)}^+ \Delta_4(m, n, p, q) \\ & + 2v_1(\omega_{(0)}^- \omega_{(0)}^+ h_{1(0)} + \omega_{(n)}^- \omega_{(0)}^+ h_{1(n)} + \omega_{(0)}^- \omega_{(n)}^+ h_{1(n)} + \omega_{(n)}^- \omega_{(n)}^+ h_{1(0)}) \\ & + \frac{v_1}{\sqrt{2}} \omega_{(m)}^+ \omega_{(n)}^- h_{1(p)} \Delta_3(m, n, p) \Big\}, \end{aligned} \quad (28)$$

where we have used the usual convention of a summation over any repeated index, with $n, m, p, q \geq 1$. In addition,

$$\begin{aligned} \Delta_3(m, n, p) &= \delta_{m+n}^p + \delta_{m+p}^n + \delta_m^{p+n}, \\ \Delta_4(m, n, p, q) &= \delta_{m+n+p}^q + \delta_{m+n+q}^p + \delta_{m+p+q}^n + \delta_{m+n}^{p+q} + \delta_{m+q}^{n+p} + \delta_{m+p}^{n+q} + \delta_m^{n+p+q}. \end{aligned} \quad (29)$$

In principle, for our calculation we should recast the above expressions in terms of the mass eigenstates, which in this case are the G and a fields in Eqs. (12) and (13). However, since there is no $\omega^+ \omega^- \omega^3$ coupling, and that of $\omega^+ \omega^- Z$ is of higher order, there is no $G^+ G^- a^Z$ coupling at leading order. Therefore, it is enough for our purposes to substitute in Eq. (28): $\omega_{(0)}^\pm \rightarrow -G_{(0)}^\pm$, $\omega_{(n)}^\pm \rightarrow s_n^W G_{(n)}^\pm$, and read the G^\pm coupling directly. Note that T_{0000} is exactly the same as that of the SM, but all the other T_{mnpq} amplitudes are $O(m_W^2 R^2)$, since they contain at least two s_n^W . However, it is possible to show that the corrections from the longitudinal gauge sector to the tree level unitarity bounds on the Higgs mass are, indeed $O(m_W^4 R^4)$. Indeed, the dominant terms at $s, t \rightarrow \infty$ are given by the quartic couplings, because the other diagrams are suppressed by h_1 propagators, thus:

$$\begin{pmatrix} T_{0000} & T_{0010} & T_{0001} & T_{0011} & \cdots \\ T_{1000} & T_{1010} & T_{1001} & T_{1011} & \cdots \\ T_{0100} & T_{0110} & T_{0101} & T_{0111} & \cdots \\ T_{1100} & T_{1110} & T_{1101} & T_{1111} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \xrightarrow{s \rightarrow \infty} \frac{-i4m_{h_1}^2 G_F}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & (s_1^W)^2 & \cdots \\ 0 & (s_1^W)^2 & (s_1^W)^2 & 0 & \cdots \\ 0 & (s_1^W)^2 & (s_1^W)^2 & 0 & \cdots \\ (s_1^W)^2 & 0 & 0 & 3(s_1^W)^4/2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (30)$$

which, for small R , has a characteristic polynomial (in the $N \times N$ case), $(1 - \lambda)(\lambda^{(N-1)} + O(m_W^2 R^2)\lambda^{(N-2)} + \dots) + O(m_W^4 R^4)\lambda^{N-2} = 0$, and, hence, the largest eigenvalue is $-i4m_{h_1}^2 G_F/(\sqrt{2})(1 + O(m_W^4 R^4))$, the others are $O(m_W^2 R^2)$ or zero. Therefore, only considering states of the Kaluza–Klein gauge sector, the corrections to the tree level SM unitarity bounds are $O(m_W^4 R^4)$. That is less than 10^{-6} for models where $R > 3 \text{ (TeV)}^{-1}$ [5,6]. Even in the case when all fields live in the bulk [7], when R can be as small as 300 GeV, the corrections from these states could not be larger than 1%.

As a matter of fact, the full unitarity analysis should be carried out also with the $Z_{(0)}$, $h_{1(0)}$ and their KK excitations, as well as the $a_{(n)}^V$ fields. However, all other matrix amplitudes for two-gauge-boson scattering have the same structure so that we find again a very tiny correction to largest eigenvalue from the pure Kaluza–Klein gauge sector blocks. The amplitudes involving Higgs or $a_{(n)}^V$ fields are not calculated with the ET and lie beyond the scope of this Letter.

5. One Higgs in the bulk and one on the brane

Let us now study the complete potential in Eq. (2), with the scalar field Φ_1 in the bulk and Φ_2 on the brane, using the full formalism and notations given in Section 2. Once more, as an application of the ET, we will study the $\widehat{W}_{L(m)}^+ \widehat{W}_{L(n)}^- \rightarrow \widehat{W}_{L(p)}^+ \widehat{W}_{L(q)}^-$ amplitude, which, again, is related to $T_{mnpq} = \widehat{G}_{(m)}^+ \widehat{G}_{(n)}^- \rightarrow \widehat{G}_{(p)}^+ \widehat{G}_{(q)}^-$.

As in the previous cases, the dominant unitarity violation in the $s, t \rightarrow \infty$ limit is given by the quartic $\widehat{G}_{(n)}^\pm$ couplings from Eq. (2). They are obtained by rewriting $\pi^\pm, \omega_{(n)}^\pm$ in terms of $G_{(n)}^\pm$, inverting Eqs. (12) and (13), and then using $\mathbf{G}^\pm = Q_W \widehat{\mathbf{G}}^\pm$. This amounts to the following substitutions:

$$\begin{aligned} \pi^\pm &\rightarrow \frac{q_0}{m} \widehat{G}_{(0)}^\pm + \frac{q_1}{m} \widehat{G}_{(1)}^\pm + \dots, \\ \omega_{(0)}^\pm &\rightarrow -\frac{d_0 q_0}{d_0^2 - m_{W(0)}^2} \widehat{G}_{(0)}^\pm - \frac{d_0 q_1}{d_0^2 - m_{W(1)}^2} \widehat{G}_{(1)}^\pm - \dots, \\ \omega_{(n)}^\pm &\rightarrow \sqrt{2} s_n^W \left[\frac{d_n q_0}{d_n^2 - m_{W(0)}^2} \widehat{G}_{(0)}^\pm + \frac{d_n q_1}{d_n^2 - m_{W(1)}^2} \widehat{G}_{(1)}^\pm + \dots \right]. \end{aligned} \quad (31)$$

In this way we have reexpressed the potential in terms of the would-be GB: $\widehat{G}_{(0)}^\pm, \widehat{G}_{(1)}^\pm, \dots$. Note that we are not interested in quartic couplings with $\widehat{G}_{(-1)}^\pm$ because it is not a would-be GB and is not “eaten” by any longitudinal gauge boson. After some tedious but straightforward calculations, we find, up to $O(m_W^2 R^2)$:

$$\begin{pmatrix} T_{0000} & T_{0010} & T_{0001} & T_{0011} & \dots \\ T_{1000} & T_{1010} & T_{1001} & T_{1011} & \dots \\ T_{0100} & T_{0110} & T_{0101} & T_{0111} & \dots \\ T_{1100} & T_{1110} & T_{1101} & T_{1111} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \xrightarrow{s \rightarrow \infty} -i \begin{pmatrix} A q_0^4 & B q_0^3 q_1 & B q_0^3 q_1 & C q_0^2 q_1^2 & \dots \\ B q_0^3 q_1 & C q_0^2 q_1^2 & C q_0^2 q_1^2 & 0 & \dots \\ B q_0^3 q_1 & C q_0^2 q_1^2 & C q_0^2 q_1^2 & 0 & \dots \\ C q_0^2 q_1^2 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (32)$$

where,

$$A = \frac{4\lambda_1 d_0^4}{[d_0^2 - m_{W(0)}^2]^4} + \frac{4\lambda_2}{m^2}, \quad B = \frac{4\lambda_2}{m^2}, \quad C = \frac{4\lambda_2}{m^2} + \frac{2\lambda_1 c_\beta^4}{m^4 s_\beta^8} + O(m_W^2 R^2). \quad (33)$$

The largest eigenvalue (compare with Eq. (27)) is now given by:

$$\begin{aligned}
& Aq_0^4 + \frac{2B^2}{A}q_0^2 \sum_{i=1}^{\infty} q_i^2 + O(m_W^4 R^4) \\
& \simeq \frac{4G_F}{\sqrt{2}} [m_{h_1}^2 c_\beta^2 + m_{h_2}^2 s_\beta^2] \left\{ 1 + \frac{2\pi^2 s_\beta^4 c_\beta^4}{3} (m_W R)^2 \frac{[m_{h_1}^2 - m_{h_2}^2]^2}{[m_{h_1}^2 c_\beta^2 + m_{h_2}^2 s_\beta^2]^2} + O(m_W^4 R^4) \right\}.
\end{aligned} \tag{34}$$

Hence, in the general case we find that the strong cancellation of the simple cases studied before, which are recovered in the $s_\beta \rightarrow$ and $c_\beta \rightarrow 0$ limits, does not occur, and, unless $m_{h_1} = m_{h_2}$, there is an $O(m_W^2 R^2)$ modification to the SM result from the pure gauge sector.

6. Conclusions

In this Letter we present a generalization of the equivalence theorem between longitudinal gauge bosons and Goldstone bosons to the case when there is one extra dimension and the Standard Model gauge symmetry is spontaneously broken by a Higgs field in the bulk and another one on the brane. The main difficulty is the identification of the would be Goldstone bosons, which are a mixture of the familiar Goldstone bosons with their own Kaluza–Klein excitations and those of the fifth component of the gauge bosons.

The equivalence theorem turns out to be a powerful tool to obtain simple expressions involving longitudinal gauge bosons as we have illustrated by calculating their scattering amplitudes in several cases. The ET has allowed us to show that the modifications from pure longitudinal gauge boson scattering to the tree level unitarity bounds of the SM are generically small and in the one Higgs limiting cases can suffer from even stronger cancellations.

Our results open up the possibility to tackle the full matrix needed for the complete unitarity violation study, including also amplitudes involving Higgs fields [13].

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